QUANTITATIVE RISK ANALYSIS OF NATURAL AND TECHNOLOGICAL DISASTERS: GENERAL APPROACH, OPTIMAL MANAGEMENT AND A PRACTICAL REALIZATION.

Ivanov L. M., T. M. Margolina and O. V. Melnichenko

Marine Hydrophysical Institute
of the Ukrainian National Academy of Sciences,

Kapitanskaya 2, Sevastopol, Ukraine, 335000
Tel.: 83-0690-520450 / Fax: 83-0690-444253 / E-mail: ocean@mhi2.sebastopol.ua

Abstract

If the evolution of an extreme event can be described by a Markovian diffusion approximation and the evolution equations can be reduced to a system of ordinary differential equations, a method for the prediction and management of natural and technological disasters is developed. In this approach three quantitative characteristics of a disaster, such as, the probability of realization, the mean time to its realization and the dispersion of the mean time are calculated. In addition, the problem of the synthesis of a stochastic optimal control of disaster risk, described by nonlinear probabilistic criteria, has an unique solution and an optimum management can be reached. An application to the problem of the radioactive damping in the Kara Sea illustrates the theory.

The key words: risk analysis of disasters, optimum management, the radioecological disaster in the Kara Sea

Introduction

The quantitative risk-analysis of natural and technological disasters and the management of these risks are of considerable interest in modern science and it can have applications for the prediction of earthquakes, ecological disasters, the study of mental disorders and revolts of prisoners etc. In recent years, these methods have been applied for the study of such exotic phenomena as the behavior of stockbrokers, alcohol influence on car drivers and censorship politics relative to the pornographic literature. The principle goal of the present paper is to discuss basic methods for the calculation of quantitative risk characteristics for natural and technological disasters. These methods are not limited by the study of some specific phenomenon but can be applied in differential domains of modern natural sciences, ecology, sociology, medicine and etc.

In our opinion, the most total and mathematically confirmed risk analysis of disasters has been developed in the catastrophe theory of *Thom* [1972], [1975], *Zeeman* [1977], *Arnold* [1983]. They defined a catastrophe as spasmodic changes of intrinsic system parameters generated in the form of sudden responses to smooth changes of external conditions (control parameters). From the mathematical point of view, the catastrophe theory is one of the possible approaches for the solution of the problem of a dynamical system strolling about stationary positions (*Pontryagin, Andronov and Vitt* [1933]) and based on the theory of smooth mappings and bifurcations. There are lot of successful applications of the theory in natural sciences, ecology, sociology and etc.

However, in spite of this, the catastrophe theory has some principal defects. First, it can be applied only for the study of so-called gradient systems or systems reduced to such type.

Second, the theory can not describe the details of system transitions between different stationary positions. Third, in reality, control parameters can change nonsmoothly, for example, stochastically. The catastrophe theory becomes useless if the number of control parameters is more than 4 (e.g., *Ebeling and Engel-Gerbert* [1980]). In addition catastrophe theory needs the development of special approaches for optimal control of stochastic objects, with a goal of determination of an exact control law in the case of arbitrary nonlinear dependencies for optimal criteria, based on the distribution density of system position vectors. As far as, we know, such optimization problem have not yet been solved.

The difficulties of the catastrophe theory discussed above can be avoided if the analogy between the disaster evolution and the process of time-first passage boundary of a stochastic trajectory is used. In this case, the quantitative characteristics for description of disaster evolution are introduced. If additionally the Markovian properties of a stochastic trajectory are postulated, then high-effective mathematical methods for the calculation of these characteristics can be developed. It will be demonstrated in the present paper. The basic theory will be illustrated by the study of the risk analysis of the radioactive disaster in the Kara Sea. This can be caused by the collision of an iceberg, generated in the summer from the Novaya Zemlya's glacier, and a container, filled with the liquid or the solid radioactive waste which had been dumped in the Kara Sea, by the former USSR, between 1960 and 1980.

Quantitative description of a natural and technological disaster.

Let to describe the system positions as a set of N characteristics $x_n(t), n=1,\dots,N$, evolution of which satisfies the following dynamical equations:

$$\frac{dx_n}{dt} = f_n(x_m,\theta_k,t),\ m=1,\dots,N\ k=1,\dots,K \qquad (1)$$

$$x_n\Big|_{t=t_o} = x_n^o \qquad x_n^o \in V \qquad (2)$$

where fk is a nonlinear operator, $\theta_k = \langle\theta_k\rangle + \theta_k'$ are changing control parametries with the statistical characteristics: the mean value $\langle\theta_k'\rangle = 0$ and the correlation function $\langle\theta_i'(t)\theta_k'(t')\rangle = \kappa_{id}\eta_l(t-t'), \eta(0)=1$, x_i^o is an initial system position, the matrix κ_{id} characterizes external noises.

It is suggested that a disaster was realized if a system passes from a position x_n to an anomal position x_n^a. Note, that in numerous practical applications the function x_n^a depends on the time and changes by some known law.

The following characteristics are introduced for the description of disasters:

the probability that a disaster happens during the time $t-t_o$ — $P(t_o,x_n^o,t-t_o)$;

the mean time of the dynamical system transition from the position x_n^o to the position x_n^a — $\langle\tau(x_n^o)\rangle$;

the dispersion of this time — $\langle\delta\tau^2(x_n^o)\rangle$.

Our purpose is to develop quantitative methods for the calculation of these characteristics and to determine the necessary conditions to which, in this case, the control parameters θ_k and the structure of nonlinearity in the equations (1) should satisfy.

It is clearly, that such a mathematical approach to the problem of disaster description can be reduced to the study of stochastic trajectory hitting into some domain, the geometrical center of which moves in the space of the system position. Details of such a problem have been discussed by *Ivanov, Kirwan and Melnichenko* [1994].

Appling their results for the calculation of P, $<\tau>$ and $<\delta\tau^2>$ these functions can be defined as:

$$\langle\tau(x_n^o)\rangle=\int_{t_o}^{\infty}(t-t_o)\partial P(t_o,x_n^o,t-t_o)/\partial t\,dt, \quad (3)$$

$$\langle\delta\tau^2(x_n^o)\rangle=\int_{t_o}^{\infty}(t-t_o)^2\partial P(t_o,x_n^o,t-t_o)/\partial t\,dt-<\tau(x_n^o)^2>, \quad (4)$$

$$P(t_o,x_n^o,t-t_o)=2\varepsilon^{n-2}\int_{t_o}^{t_o}\beta(t')p[t_o,x_n^o,t',x_n^a(t')]dt'+O(\varepsilon^{n-2}). \quad (5)$$

Here, ε determines a geometrical size of domain separated around the point $x_n^a(t)$. It was considered with the aim of excluding of situations when a dynamical system can multiply pass into the domain and leaves it without any disaster changes. The function p is the density probability of dynamical system transition from the position x_n^o to the position x_n^a. This function satisfies the Fokker-Plank type equations.

The function $\beta(t)$ can be calculated as:

$$\beta(t)=\int_{S_\eta}\frac{\partial\varphi}{\partial\gamma}dS_\eta \quad (6)$$

where, $\partial/\partial\gamma$ is a normal derivitative determined at the surface $S\eta$; the function φ is the solution of the following boundary problem:

$$\Delta\varphi=0 \quad (7)$$

$$\varphi\Big|_{S_\eta}=1 \quad (8)$$

$$\varphi\to 0 \quad \text{if} \quad \eta\to\infty \quad (9)$$

Here, Δ is n-dimensional Laplace's operator; η are the variables for which, the matrix κ_{kl} reduces to the unit matrix.

It was demonstrated by *Ivanov, Kirwan and Melnichenko* [1994], that if the $S\eta$ is a n-dimensional ellipsoid described by the following equation:

$$\alpha_n\,\eta_n^{\;2}=1 \quad (10)$$

then, the $\partial\varphi/\partial\gamma$ can be calculated analytically. The result of a such calculation is:

$$\frac{\partial\varphi}{\partial\gamma}=\prod_{n=1}^{N}\alpha_n^{\;1/2}\left[\sum_{n=1}^{N}\alpha_n^{\;2}\eta_n^{\;2}\right]^{-1/2} \quad (11)$$

Let us discuss the limitations on a choice of the control parametries θ_k and the functions f_n .

First, the equations (3) and (4) are true for arbitrary nonlinear functions θ_k and f_n . Second, the deduction of equation (5) requires the Markovian diffusion approximation for the description of system positions. Therefore, we should suggest that the f_n which depends linearty from $\theta_k^{'}$ and $\theta_k^{'}$ is the Gaussian white noise, or the Gaussian colour noise, reduced to first noise type by the method of arbitrary correlation time (*van Kampen* [1970]). In more modern version , this method are discussed by *Pugachev and Sinitsin* [1996]). Note, that in other respects the f_n is an arbitrary nonlinear function.

Thus, for the quantitative description of disasters we introduced three characteristics: P, $<\tau>$ and $<\delta\tau^2>$ calculated when x_n is the Markovian diffusion process. From the theoretical point of view, it limits the types of possible disasters which can be described by our approach. However, in practice, such limitations are not essential. In addition, it will be demonstrated in the next section of the paper, that for the approach discussed above, the synthesis problem of stochastic control of a disaster risk can be formulated and solved.

Calculation of quantitative characteristics of disasters.

It results from the equations (3)-(5), that the determination of $<\tau>$, $<\delta\tau^2>$ and P can be reduced to the calculation of the density probability of system transitions from the positions x_n^o to the positions x_n^a. Such density probability satisfies the Fokker-Plank type equations:

$$\frac{\partial p}{\partial t} = L(p) \tag{12}$$

$$p\Big|_{t=t_o} = \delta(x_n - x_n^{'}) \quad p\Big|_{x_n \to \infty} \to 0 \tag{13}$$

Here $L = \frac{1}{2}\nabla_n \kappa_{nl} \nabla_1 - <\theta_n> \nabla_n,\ \nabla_n = \frac{\partial}{\partial x_n}$ where, in accordance with the limitations on the choice of f_n discussed above, $f_n = <\theta_n> + \theta_k^{'}$.

The equation (12) with the initial and boundary conditions (13) is a linear n-dimensional parabolic equation. There are its analytical solutions obtained only for some special forms of θ_n and κ_{nl} (see, *Haken* [1985], *Klyatskin* [1980], *Dimentberg* [1983]). In the general case, if $n \leq 3$ the finite-differences methods are used for its solution and if $n > 3$, the solution can be reached by the method of Gaussian approximation or the Monte-Carlo technique. The last methods are low-effective when the number of control parameters is very large.

In this section a high-effective numerical method for the solution of (12) will be developed. It bases on the adaptation of the method of small parametry for the numerical solution of differential equations suggested by *Dorodnitsin* [1973].

Let us discuss the method in more detail. Consider some parameter ε changing from 0 to 1 and rewrite the equations (12) in the following form:

$$\frac{\partial p}{\partial t} = L^{*}(p) + \varepsilon\left[L - L^{*}\right](p) \tag{14}$$

Here L^* is the specially choosen operator, such that if ε is equal to 0, the equation (14) has a simple analytical or numerical solution. Note, that if ε tends to 1, the equation (14) is transformed to the equation (12).

Add to equation (14), the equation for the function $\Psi = \frac{\delta p}{\delta \varepsilon}$, which can be written as:

$$\frac{\partial \Psi}{\partial t} = L^*(\Psi) + \varepsilon \frac{\delta [L - L^*]}{\delta \varepsilon} \Psi + [L - L^*](p) \quad (15)$$

Now, divide the interval of ε change into I equal parts and apply the following iterative procedure for the solution of the equations (14) and (15):

$$p_{[i+1]} = p_{[i]} + \Delta\varepsilon \Psi_{[i]} \quad (16)$$

$$\frac{\partial \Psi_{[i]}}{\partial t} = L^*\left(\Psi_{[i]}\right) + \varepsilon_i \frac{\delta L^*}{\delta p} \Psi_{[i-1]} + [L - L^*]\left(p_{[i]}\right) = 0 \quad (17)$$

$$\varepsilon_i = \Delta\varepsilon\, i, \quad i = 0, \ldots, I-1 \quad (18)$$

In the case, p_I approximates a true solution of (12) with an accuracy equaled to $O(\Delta\varepsilon)$. If ε tends to 1, the iterative procedure (16)–(18) converges linearly.

The zero iteration can be found as a solution of the following equations:

$$\frac{\partial p_o}{\partial t} = L^*(p_o) \quad (19)$$

$$p_o\Big|_{t=t_o} = \delta\left(x_n - x_n^o\right), \quad p\Big|_{x_n \to \infty} \to 0 \quad (20)$$

$$\frac{\partial \Psi_o}{\partial t} = L^*(\Psi_o) + [L - L^*](p_o) \quad (21)$$

$$\Psi_o\Big|_{t=t_o} = 0, \quad \Psi_o\Big|_{x_n \to \infty} \to 0 \quad (22)$$

The Green's function of the operator $\partial/\partial t + L^*$ was used for determination of the functions $\Psi_o, \Psi_1, \ldots, \Psi_{I-1}$. In the case, the solution for each iteration has been presented as a multidimensional space integral calculated analytically or numerically by reduction to set of n independent single dimensional integrals.

It was demonstrated by our experience that, just in the case when $\Delta\varepsilon = 1/2$, the solutions obtained from (12) are suitable for the practical estimations. Note, that the function p calculated by our method is essentially different from the Gaussian distribution function used usually in n-dimensional cases.

Management problem of disaster risk.

Let us discuss the synthesis problem adapted for management of the disaster risk. From the mathematical point of view, it can be formulated as: determine the control vector $\langle \theta_k^{opt} \rangle$ which minimizes the following probabilistic functional:

$$J = \int_{t_o}^{t^*}\int_{V}\left\{P^2 + \left[\langle \theta_k \rangle - \langle \theta_k^o \rangle\right]^T D_{kl}\left[\langle \theta_l \rangle - \langle \theta_l^o \rangle\right]\right\} dt\, dx_n^o \quad (23)$$

Here Dkl and $<\theta_k^o>$ are both known the quadratic symmetric matrix and control vector, respectively.

To discuss the physical interpretation of the cost function (23). The first item in the integrand of (23) is a mathematical reflection of the fact that disaster risk can be decreased by the minimizing of $<\delta\tau^2>$ and the maximizing of $<\tau>$. The requirement to reach minimum departure from a some given control vector $<\theta_k^o>$, determinated by financial possibilities of technical realization of a control, reduces to the second item in the integrand.

In principle, the functional (23) can be minimized by the method of dynamic programming (in detail, see *Sirazetdinov* [1977] and *Sokolov* [1996]). In this case $<\theta_k^{opt}>$ can be analytically found (see, *Sokolov* [1996]). But it depends on several auxiliary functions which are solutions of a system of n-dimensional quasilinear probabilistic equations. These equations can be only solved in special cases which are not of interest in practice.

In our opinion, a simple optimum control should be found, if the iterative procedure discussed above is used, for the minimizing of (23). Using the basic principle of dynamic programming (the Bellman's optimum principle) (see, *Bellman* [1957]) and the iterative method for the determination of P, the following iteration procedure for the calculation of the $<\theta_k^{opt}>$ can be written as:

$$<\theta_k^{opt}>=\lim_{i\to I}\left(<\theta_k^{opt}>\right)_{[i]} \tag{24}$$

$$\left(<\theta_k^{opt}>\right)_{[i]}=<\theta_k^o>+D_{kl}^{-1}\left[\int_{t_o}^{t_*}\int_V G\frac{\partial P_{[i-1]}}{\partial x_k}dx_n\,dt\right]^T \tag{25}$$

$$\left(<\theta_k^{opt}>\right)_{[o]}=<\theta_k^o> \tag{26}$$

where G is the Green's function of operator $\frac{\partial}{\partial t}+L^*$, L^* does not depend on $\left(<\theta_k^{opt}>\right)_{[i]}$.

Note that for the determination of θ_k^{opt}, linear parabolic equations should only be solved.

Application

The principal theoretical approach developed in the present paper is illustrated by the calculation of the probability of possible a radioactive disaster in the Kara Sea. It is well-known, that the former USSR dumped in the Sea liquid and the solid radioactive wastes with total activity approximately equaled to 90 PBq for the period from 1960 to 1980. Containers with radionuclide wastes were being dumped at the Novaya Zemlya's shallow shelf where the depths change from 10 m to 40 m. In summer, the Novaya Zemlya's glacial creep in the Kara Sea can generated numerous differential-scale icebergs. The icebergs move in different directions. This drift is generate by the marine circulation and the wind. Therefore, an iceberg can bump into a dumped container and destroy it. Then, large quantitatives of Cs-137, Sr-90 and other radionuclides can be dispersed by the marine circulation and diffusion over a vast geographic region.

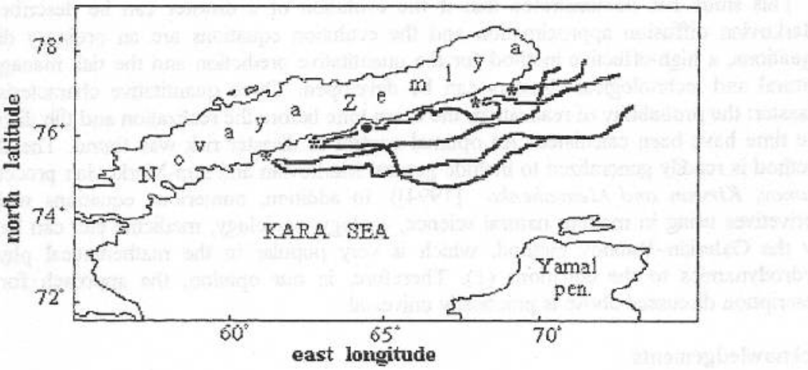

Fig. 1. Map of the geographic region where the container with radioactive wastes were dumped. The bullet is the position of dumped container in the Stepavogo Bay. The asterisks indicate to the places where iceberg generation is possible. The bold solid lines are the possible iceberg trajectories.

Our goals are to estimate the probability of such disaster and to calculate the mean time before possible depressurization of the container, dumped in the Stepavogo Bay. The container position and the places where iceberg generation is possible have been indicated in Fig. 1.

The dynamics of iceberg drifting by the action of marine circulation and wind are described by the model of *Chari* [1979]. The model was benefitted the prediction of iceberg pathways in the Arctic and the Southern ocean (*Banke and Smith* [1984], *Elisov* [1995]). It was postulated that the real circulation in the Kara Sea can be separated into two parts: the climatic flow calculated by dr. M. Kulakov from the climatic salinity and temperature, and the stochastic adding, the dispersion of which was determinated from the results of direct measurements of the large-scale circulation and the tidal characteristics contained in the data bank of the Arctic and Antarctic Research Institute (*Regional Data Bank "Contamination of the Arctic Ocean Basin* [1994]). Similarity, the climatic and stochastic components of the wind have been separated.

In our study the depth of the submarine portion (h) of an iceberg changes from 5 m to 40 m. In addition, initial iceberg positions were variable. There are three variants in the calculations: initial iceberg positions were determined with coordinates $\varphi=75°00'$ N, $\lambda=61°10'$E and $\varphi=76°20'$ N, $\lambda=68°00'$E and an iceberg can be with equal probabilities in one of four positions with coordinates:

$\varphi_1=75°00'$ N, $\lambda_1=61°10'$E; $\varphi_2=75°20'$ N, $\lambda_2=62°40'$E;

$\varphi_3=75°50'$ N, $\lambda_3=65°40'$E; $\varphi_4=76°20'$ N, $\lambda_4=68°00'$E.

The results of these calculations are discussed briefly below.

The distruction of the container dumped in Stepavogo Bay by an iceberg is not unlikely event. The probability of this event, calculated for one century is approximately equal to 0.033. The minimum possible time for the disaster realization calculated, when h=20m, is less than 60 days. Therefore, we can confirm, that not only the chemical corrosion of the containers with the liquid and the solid radioactive wastes dumped in the Kara Sea but also their collision with icebergs can be a reason of a radioecological disaster in the Kara Sea.

Conclusions

This study has demonstrated that if the evolution of a disaster can be described by the Markovian diffusion approximation and the evolution equations are an ordinary differential equations, a high-effective method for the quantitative prediction and the risk management of natural and technological disasters can be developed. Three quantitative characteristics of a disaster: the probability of realization, the mean time before the realization and the dispersion of the time have been calculated. An optimal control of disaster risk was found. The developed method is readily generalized to include general Markovian and non-Markovian processes (see, *Ivanov, Kirwan and Melnichenko* [1994]). In addition, numerious equations with spacial derivetives using in modern natural science, ecology, sociology, medicine etc. can be reduced by the Galerkin–Bubnov method, which is very popular in the mathematical physics and hydrodynamics to the equations (1). Therefore, in our opinion, the approach for disaster description discussed above is practically universal.

Acknowledgements

The bulk of the research and the primary support of L. M. Ivanov, O. V. Melnichenko and T. M. Margolina has come for the Arctic and Antarctic Research Institute (St. Peterburg). We thank Dr. M. Yu. Kulakov who kindly provided us the ST data and the results of circulation calculation in the Kara Sea. In addition we are grateful to anonymous reviewer(s) for useful criticism and corrections.

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