Calculation of extreme tidal sea levels based on nonlinear programming methods

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ABSTRACT

A numerical method is proposed for calculation of extreme tidal levels from the harmonic constants of any number of component waves with the aid of nonlinear programming. The method is applied on a computer for 34 component waves obtained from harmonic analysis of a monthly observation series by the Doodson method. The influence of the number of component waves considered on the extreme tidal levels is investigated.

1. INTRODUCTION

Information on extreme sea levels is needed for solution of various practical problems in marine navigation and hydraulic construction. On the tidal seas of Russia, where the mean tidal range (the difference between successive high- and low-water levels) is 50 cm or more, depths must be reduced to a theoretical zero depth (TZD). The theoretical zero depth is calculated by reducing the initial mean level by the amount of the largest tidal ebb amplitude that is astronomically possible, which is determined for each station from the harmonic constants. Many papers have been devoted to determination of extreme tide heights. The Laplace method is used for the particular case of regular semidiurnal tides [Vladimirskii 1941]. Until recently Vladimirskii method [Vladimirskii 1936] has been used in practical hydrographic and oceanographic studies to calculate extreme tide neights. In this method, the TZD is usually calculated from the harmonic constants of the eight principal components of the tide waves. Vladimirskii's method is extremely time-consuming. It does not take account of secondary tidal waves. Although their amplitudes are relatively small, they may, taken together, have a perceptible influence on the level.

In 1956, Kudryavtsev proposed a method for determining the TZD from the harmonic constants of the eight principal component waves of the tide. This is essentially a simplified Vladimirskii method. The author assumed that the tide waves with the highest amplitudes are also in phase. This assumption is quite artificial. The study [Peresypkin 1966], in which the extreme tide heights are calculated from the harmonic constants of 30 component tide waves, is similarly deficient.

It has recently become possible to use computers to solve the problem for any number of component waves. In 1974, Peresypkin proposed a method for finding extreme tide heights, which he developed in detail and used on a computer for 13 tide-wave components [Peresypkin 1974]. The method is based on solution of a system consisting of four equations with four unknowns. The deficiencies of this method include errors associated with the expansion in Taylor series and the poor convergence of the iterative process in the case of shoal waves with high amplitudes, which makes it necessary to carry out the solution in several steps.

2. NUMERICAL METHOD

The method proposed here differs from the above methods in that it is not subject to these shortcomings and uses simple computations that do not require a large computer memory. The method is developed and applied on a computer for 34 component tide waves. However, it can easily be extended to any number of waves, and an increase in the number of waves considered has no significant effect on the volume of a computational work. The problem is formulated as follows. The harmonic constants of the tide at some station are given. It is required to determine the highest and lowest levels that are theoretically astronomically possible at this station. The sea level H_t relative to mean sea level can, as we know, be represented in the form of a sum of tide waves:

$$H_t = \sum_{k=1}^{n} f_k H_k \cos\varphi_k ,$$

where for each wave f_k is a reduction factor that depends on the longitude N of the moon's ascending node, H_k is the amplitude harmonic constant, and φ_k is the phase of the tide component wave.

Harmonic analysis of tide observations consists in breaking up the composite wave into its individual components. Our problem is therefore to synthesize the component waves in such a way that the height of the tide assumes extreme values. According to [Vladimirskii 1941], it is necessary to shoose reduction-factor values that will give the largest effect for H_{min} and H_{max}. Reduction factors corresponding to $N = 0°$ are chosen for the diurnal tides and values corresponding to $N = 180°$ for the semidiurnal tides. In the case of mixed tides, the calculations are made in both ways and the largest absolute values are chosen. Our method is designed to use initial data in the form of results of harmonic analysis of monthly series of observations by the Doodson method, which can be made to yield the harmonic constants of 34 waves. Calculations that consider larger numbers of secondary waves with amplitudes in the centimeters and fractions of a centimeter do not promise any significant improvement. This becomes understandable when it is remembered that the absolute variations of the harmonic constants of the main waves exceed the harmonic constants of some of the secondary waves.

The values of the reduction factors for the cases N = 0° and N = 180° and the expressions for the phases as functions of the principal astronomical elements of the 34 component tide waves (2 long-period, 10 diurnal, 10 semidiurnal, 12 shallow-water) are chosen from [Peresypkin 1966], where t is the mean civil time reckoned from midnight, h is the mean longitude of the sun, s is the mean longitude of the moon, p is the mean longitude of lunar perigee. The rest of fundamental astronomical elements have the following periods, during which they assume all possible values from 0 to 360°: t = 24 mean hours, s = 27,3 mean days, h = 365,25 mean days, p = 8,85 years, N = 18,63 years. Since the periods are not commensurable with one another, we can, over an indefinitely long span of time, obtain simultaneous combinations of all combinations of values of t, s, h and p, while N can assume the constant values 0 and 180°. The astronomical conditions are determined from the following criteria: $s - h \approx 0°$ at new moon, $s - h \approx 180°$ at full moon, $s - p \approx 0°$ at perigee, $s \approx 0°$ (or 180°) for the moon on the equator, $s \approx 90°$ for the greatest northern declination of the moon, and $s \approx 270°$ for the greatest southern declination of the moon.

For simplicity in the exposition that follows, we introduce new notation, namaly: we shall denote a function by ψ and the argument corresponding to it by a vector x

$$x = (x^{(1)}, x^{(2)}, x^{(3)}, x^{(4)}) = (t, h, s, p).$$

In the new notation, the problem can be formulated thus:

$$\min(\max)\,\psi(x) = \min(\max) \sum_{k=1}^{34} f_k H_k \cos\varphi_k. \quad (1)$$

The problem (1) is a problem of nonlinear programming. At this time, one of the most widely used methods for finding the extremes of a function is the gradient method with its various modifications. This method is simple and makes possible complete solution of the problem in many cases. However, it has a highly important shortcoming: it can be used to find only local extremes of a function. In practice, this difficulty can be overcome by preliminary investigation of the function and subsequent comparison of the results obtained. We propose here an extreme-search method that combines the method of sequential enumeration and the method of steepest descent.

Let the function $\psi(x)$ be defined on a compact set

$$X \in E_4,\ E_4 = \{x^{(1)}, x^{(2)}, x^{(3)}, x^{(4)}\},$$

where E_4 is a four — dimensional Euclidean space. The maximum of the function $\psi(x)$ on X is reached on a certain nonempty set $S \subset X$. We note first of all that $\psi(x)$ is continuous on E_4 and 2π periodic with respect to each argument, i.e., $X \in [0,2\pi]$. We denote the solution of this problem by $F(x_*)$. It is necessary to determine $F(x_1)$, the extreme of the function of four variables, with a predetermined error ε and to find the point x_1 at which this approximation is satisfied:

$$F(x_*) - F(x_1) < \varepsilon .$$

The problems of finding the lowest and highest levels are solved similarly. For consistency, we introduce the symbol

$$L = \begin{cases} 1, \text{if } \max \psi(x) \text{ is sought,} \\ -1, \text{if } \min \psi(x) \text{ is sought.} \end{cases}$$

The object of the search is to find a combination of values of the arguments x at which the value of the function $\psi(x)$ is at maximum (by introducing the symbol L, we have reduced the extreme-search problem to a maximum-search problem).

The numerical solution of Eq. (1) is carried out in two steps. In the first step of the extreme-level search, a discrete minimax problem is solved to find the initial approximation. We vary successively only the first coordinate of the vector x with the other arguments fixed, assuming

$$x_1^{(1)} = 0,\ x_2^{(1)} = x_1^{(1)} + C_1, \ldots,$$

$$x_n^{(1)} = x_{n-1}^{1} + C_1,$$

where C_1 is a certain increment (const). We find F_n from the formula

$$F_n = L \max [L\psi(x_1), L\psi(x_2), \ldots, L\psi(x_n)]. \qquad (2)$$

If we have obtained $x_n^{(1)} > 2\pi$ after the n-th step, the level calculations at this step are terminated, with the result that F_n assumes the maximum value. In the case of semidiurnal tides, when the largest tides are observed at new moon and full moon, the values of the other three coordinates in the calculations with (2) can be put equal to 180° (new moon,

perigee), while in the case of

diurnal tides, when the highest tides are observed at the greatest declinations of the moon, the values of the other three coordinates are taken equal to 90° (greatest northern declination, perigee). The choice of these initial values of x_o, for which $\psi(x_0)$ is near the extreme, and the use of the enumeration procedure for the first coordinate eliminate unpromising local extremes from consideration. In determination of the lowest level of an irregular semidiurnal tide, for example, the sequential enumeration method enables us to find low water springs, and then it is precisely this level that is subsequently minimized (improved).

We fix the value of the vector x_n, which corresponds to F_n, and go on to the second step of the search, in which the method of steepest descent is used with double the change in increment [Evtushenko 1971]. Let us illustrate the working principle of the method in this case. We take the step

$$\widetilde{x}_1 = x_n + \tau_1 L \nabla \psi(x_n).$$

There $\tau_1 > 0$ is a coefficient and $\nabla\psi$ is the gradient of the function ψ, i.e., a four-dimensional vector with the coordinates $(\partial\psi/\partial x^{(1)}, \partial\psi/\partial x^{(2)}, \partial\psi/\partial x^{(3)}, \partial\psi/\partial x^{(4)})$. The gradient $\nabla\psi$ indicates the direction of steepest ascent (descent) of the function in the neighborhood of a given point. For the point x_* to be an extreme point of $\psi(x)$, it is necessary that the equality $\nabla\psi(x_*) = 0$ be satisfied. The magnitude of each i-th coordinate of the gradient vector of $\psi(x)$ is evaluated from the formula

$$\frac{\partial \psi}{\partial x^{(i)}} = \frac{\psi(\widetilde{x}) - \psi(x)}{\Delta x};$$

here $\widetilde{x}^{(j)} = x^{(j)}$ for all $j \neq i$ and $\widetilde{x}^{(i)} = x^{(i)} + \Delta x$ for $j = i$.

If the condition

$$L(\psi(\widetilde{x}_i) - \psi(x_n)) > 0 \qquad (3)$$

is satisfied at the step $i = 1$, we take $\tau_2 = 2\tau_1$ and make a second step:

$$\widetilde{x}_2 = \widetilde{x}_1 + \tau_2 L \nabla \psi(\widetilde{x}_1),$$

and so forth until condition (3) is satisfied. If condition (3) is violated at a certain i-th step, we take

$$\widetilde{x}_i = \widetilde{x}_{i-1} + L \frac{\tau_{i-1}}{4} \nabla \psi(\widetilde{x}_{i-1}).$$

The optimum step length is chosen in this way. The iterations are terminated if the condition

$$|\nabla \psi(x_i)| < C_2,$$

where C_2 is a coefficient, is violated. Thus, the search process reduces to determination of the most promising regions, in which the method of steepest descent is then used to find the value of the function $F(x_1)$ with the required accuracy.

The program generates the extreme value F, which represents the largest deviation from mean level, as well as the values of the astronomical arguments t, h, s, p, and N that correspond to the extreme. The value of t determines the mean civil time, and the value of h the day and month. These data can be used for calculations of the values of s and p at 0^h on 1 January, and then the tables of the astronomical elements can be consulted to determine the year in which the extreme tide level may occur for the particular station.

3. THE INFLUENCE OF THE TIDE COMPONENT WAVES

Table
Extreme Levels (cm)
as Functions of Number
of Waves Considered

No of waves	Ekaterininskaya Gavan		Kem	
	H_{min}	H_{max}	H_{min}	H_{max}
8	− 203	196	− 98	97
13	− 198	202	− 92	111
25	− 209	213	− 89	108
34	− 209	214	− 90	111

Table presents values computed from 8, 13, 25, and 34 component waves for the extreme tidal levels at the Ekaterininskaya Gavan and Kem stations (White Sea). The group of thirteen waves included, in addition to the eight fundamentals, the three shallow-water waves M_4, MS_4, and M_6, which are most often taken into account in computation of extreme heights, and the two long-period waves Mm and Msf. In the calculation using 25 component waves, the thirteen waves, named above were supplemented by the remaining semidiurnal and diurnal waves. The data in Table indicate that the positions of the extreme levels are adjusted as the number of secondary waves is increased, and that calculations based on 25 and 34 component waves give closely similar results. The largest difference between the values of the extreme tidal levels obtained from 8 and 34 component waves is 18 cm for Ekaterininskaya Gavan and 14 cm for Kem at high water springs. Thus, the additional waves, from the ninth on, change the results of the extreme-level determination significantly. Consequently, the proposed method makes it possible to improve the accuracy of the calculation and can be used for practical purposes.

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